3D Simulation of Plasma in Case of Synchrotron Gyromagnetic Autoresonance

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A 3D simulation for plasmas heated by microwaves and confined in a simple magnetic mirror trap in case of synchrotron gyromagnetic autoresonance (SGA) was carried out. Simulation using particle-in-cell method shows the possibility to obtain relativistic plasma with parameters (average electron energy, plasma density and dimension) which can be varied in wide ranges and to define optimal parameters of SGA. As it follows from obtained results SGA regime also permits to produce a controlled bunch of relativistic electrons. The latter property of SGA can be used as a basis for designing a compact source of hard x-ray.

Introduction

The procedure to produce relativistic plasma under the conditions of electronic cyclotron resonance (ECR) in the growing in time magnetic field called synchron gyromagnetic autoresonance (SGA) was proposed in the early 80′ [1].

In case of SGA magnetic field variation in time $B(t) = B_o[1+ b(t)]$ (where $B_o = m_o c\omega/e$, m_o and e being rest electron mass and charge respectively, ω being angle frequency of HF field and $b(t)$ is a growing function of time) counterbalances relativistic electron mass change. Consequently the resonance condition $\omega \cong \omega_{ce}$, ($\omega_{ce} = eB(t)/m_o\gamma c$, $\gamma = (1-v^2/c^2)^{-1/2}$, v being the electron's velocity) is automatically maintained in a contrast to ECR. Maximal energy obtainable by electron in the SGA mode is limited by radiation losses only, while its average kinetic energy (keV) is determined by the magnitude of magnetic field and does not depend upon the strength of HF field: $W(t) \approx 511\ [B(t)/B_o - 1]$.

Experiments on the devices using this method (plasma synchrotrons Gyrac-0, Gyrac-D and Gyrac-X) have shown possibility of generation and accumulation of relativistic plasma and electron bunches [2-4], nevertheless the parameters obtained do not allow to use SGA for practical applications.

This paper describes tri-dimensional numeric simulation of plasma in the simple magnetic mirror trap under the SGA conditions for the conventional setup of plasma synchrotron Gyrac (fig.1). Main parts of the synchrotron are: 1 – TE_{111} is a vacuum chamber, 2 are coils creating the static magnetic field, 3 are pulse magnetic coils, 4 is a target.

Numeric model uses the particle-in-cell method, that is widely used to study complex dynamic plasma entities [5, 6].

In a contrast to analytical description and study, that was done previously on a two-dimensional numeric model [3, 4], the results of tri-dimensional simulation allow most fully analyze the processes occurring in the SGA-plasma and design more perfect experimental devices employing the SGA method. Present paper studies efficiency of electron capture, evolution of SGA parameters, i.e. plasma and processes that accompany plasma electrons acceleration under the conditions of synchronous gyromagnetic resonance.

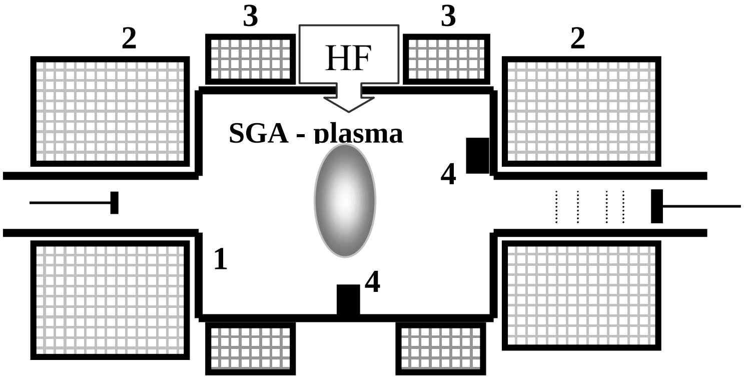

Fig. 1 Scheme of Gyrac plasma synchrotron

The purpose of the numeric experiments is to analyze the influence of initial plasma parameters, magnetic field configuration and other SGA parameters upon the electron capture into the acceleration mode, confinement of created relativistic plasma in the open magnetic trap and to determine optimal conditions for creation of SGA-plasma and relativistic electronic bunches.

1. Numeric model

One of the most efficient and commonly used methods of plasma processes simulation is the particle-in-cell method [6].

The scheme of the method used in this paper includes the following main steps:

From the known particle distribution in the X, Y, Z space at τ^n instant (n being the time step number) one finds the charge distribution in the nodes of the given lattice $\rho\ (i,j,\ k) \equiv \rho\ (i\Delta X,\ j\Delta Y,\ k\Delta Z)$, where $\Delta X,\ \Delta Y,\ \Delta Z$ are the lattice spacings in the $X,\ Y$ and Z directions, and $i = 1,\dots,\ N;\ j = 1,\dots,\ M;\ k = 1,\dots,L$. $N,\ M$ and L are the numbers of the lattice nodes in the X, Y and Z directions respectively. Charge distribution in the lattice nodes is determined by reverse bilinear interpolation (procedure of charge “spreading” among the eight nearest nodes).

Poisson equation is solved for the given lattice. Field values $E^n_{i,\,j,\,k}$ in the lattice nodes are found by taking limited-differential derivatives.

Electric field at the particle positions is a superposition of plasma’s own field, which is determined by bilinear interpolation, external HF electric filed at the τ^n instant and electric whirl field arising with the growth of magnetic field in time.

Next step is to integrate the motion equations in the Newton-Lorentz form:

$$\frac{d\boldsymbol{p}}{dt} = e\left(\boldsymbol{E}_{hf} + \boldsymbol{E}_s + \boldsymbol{E}_{ind} + \frac{\boldsymbol{v}\times\boldsymbol{B}}{c}\right) \tag{1}$$

where $\boldsymbol{p}$ is electron impulse, $\boldsymbol{E}_{hf}$ is the HF electric field strength, $\boldsymbol{E}_s$ is the plasma’s own electric field, $\boldsymbol{E}_{ind}$ is an electric whirl field arising in plasma due to the growth of magnetic field in time.

The calculations cycle is concluded by calculation of new coordinates of the particles. Thereafter this cycle is repeated.

Limited-differential analog of equation (1) in dimensionless form is as follows:

$$\frac{\boldsymbol{u}^{n+1/2} - \boldsymbol{u}^{n-1/2}}{\Delta\tau} = \boldsymbol{g}^n + \frac{\boldsymbol{u}^{n+1/2} + \boldsymbol{u}^{n-1/2}}{2\gamma^n}\times\boldsymbol{b}^n \tag{2}$$

Where $\boldsymbol{u}$ is electron impulse in m_oc units, $\boldsymbol{g}^n$ is the resultant dimensionless electric field at the n instant, $\boldsymbol{b}^n$ is magnetic field normalized over B_o, γ is the relativistic factor, $\tau = \omega t$ is dimensionless time, $\Delta\tau$ is time step.

Calculation of magnetic field created by the system of axis-symmetrical coils was performed in the nodes of the given lattice. The field at the particle locations was determined by interpolation, in a similar way to the determination of charge density in the nodes of the lattice.

TE_{111} electric field, induced in the cavity, was approximated by the $E(r,z)=E_0\cos(\beta z)$ law, where coordinate z = 0 corresponds to the cavity median plane, β is normalization coefficient, E_o is the strength of HF electric field.

Numeric model was chosen as long as it allows to study the following problems:

1. Study of the conditions for the capture of plasma electrons and the dependence of capture efficiency on the initial conditions (plasma density, strength of electric HF field, static magnetic field configuration).
2. Study of plasma evolution in SGA process.
3. Analysis of particles loss from the plasma.
4. Study of electron energy spectrum dependence upon the initial conditions.

At the instant $\tau = 0$ uniform space distribution of each type of particles (electrons and ions) was assumed in a cylinder with an axis coinciding with the axis of the vacuum chamber. Ions were assumed to have zero initial impulses. Ions were assumed to have single charges, the number of ions equal to that of electrons. Thus in the model under discussion the initial plasma was completely ionized, uniform and neutral. Note, that the model used real ion mass to electron mass ratio.

As for diamagnetic effects, at the electron energies $W \approx 1\ MeV$ in the plasma with $(1\cdot10^9-1\cdot10^{10})\ cm^{-3}$ density their influence on the SGA process is insignificant, and these are the parameters considered in the present paper. This allows to take into account in the particle method scheme the electrostatic interactions only.

Differential analog of Poisson equation was performed by cyclic reduction method with the use of fast Fourier transformation. By solving this equation periodic boundary conditions were used (chamber walls influence on the SGA process was neglected). Such an approximation is valid if the linear dimensions of the lattice are at least twice larger than characteristic dimensions of the plasma being simulated.

On the grounds of peculiarities of the processes under investigation, i.e., rapid variation of all the parameters (plasma density, cyclotronic radius of electron rotation etc.), the lattice spacing was chosen in such a way, that two main condition were fulfilled:

a) relativistic particle has to pass the distance equal to lattice spacing (lattice spacings along X, Y and Z were chosen equal) not less than in two time steps, i.e., $\Delta X = \Delta Y = \Delta Z = 2c\Delta\tau$, where $\Delta\tau$ is the integration step of motion equations $(\Delta\tau = 5\cdot10^{-3} - 8\cdot10^{-3}$ of an HF field period);
b) number of the particles in one cell has to exceed unity in a significant way.

Space limitation of the process was taken into account to analyze the longitudinal and transverse particle losses from the plasma. The boundaries, corresponding to the conditions of real experiment were introduced. The particles reached the chamber walls were considered lost.

It is noteworthy, that despite the rapid development of computer technologies, tri-dimensional plasma simulation, particularly with the account of relativism, remains “a white elephant” because of the large expenditure of computer time. Therefore in the development of the present model rather economic algorithms were used both for motion equations [5] and for Poisson equation solution [6]. Still, average calculation time of one variant for 25 000 particles of each type in case of relativistic electron bunch generation and 50 000 particles for generation of relativistic plasma on the 32 x 32 x 32 lattice appeared rather significant and for one typical case (250 000 time steps) was appr. 30 hours. Calculation time per one particle per one time step was appr. 8.6 μs. Calculations were performed on a PC with Pentium II 333 MHz processor using Fortran Power Station 4.0.

To improve the flexibility and reliability of program performance, algorithm was split into three main blocks. The first initiation block generates (or, if the calculations are to be continued, restores) space distribution of the particles and their impulses. The second block performs the numeric experiment. The third block is the one of diagnostics and final treatment of the results.

After running a certain, set by the user, number of time steps, information on particles coordinates and pulses is output as an experimental results file. If necessary, it is possible to interrupt the work of the program and, after selective processing of the results, to resume the it with the necessary changes of the numeric experiment parameters. To spare disk memory, information on coordinates and impulses of each article was saved in the binary format.

Numeric experiment allowed to obtain and process the following data:

1. Information on spatial distribution of the particles, which allowed to follow plasma evolution in time.
2. Data on particles (electrons and ions) energies for the analysis of energy spectrum variation in time, and also on the electron capture efficiency.
3. Particles losses from the plasma.
4. Dependence of the maximal value of electric field arising in the plasma on time.
5. It was made possible to resume calculation beginning from any time step, besides after changing the process parameters.

1. Results of numeric simulation

This paper studied the dependence of the capture of plasma electrons into the synchronous gyromagnetic autoresonance upon misalignment of initial magnetic field, gas type, the magnetic field growth rate, initial plasma density and mirror ratio.

Initial plasma density varied from $1\cdot10^9\ cm^{-3}$ *to* $5\cdot10^{10}\ cm^{-3}$, initial radius was chosen equal to *0.6* and *2.4 cm* and the length of plasma cylinder $l = 6\ cm$. Calculations were performed for two types of singularly ionized plasma: for hydrogen and for argon. HF electric field strength ($f = 2.45\ GHz$) varied from *600 V/cm to 5 kV/cm*. Calculation were done for two mirror ratios: $R = 1.2$ and $R = 2.0$.

Analysis of evolution of the spatial plasma distribution and its parameters has shown the possibility to create within the initial cold (non-relativistic) plasma by means of SGA various objects, that can be brought to two limiting cases, shown in Figs. 2 and 3. Calculations were done for the following parameters: $E = 3kV/cm$, argon plasma, mirror ratio $R = 1.2$, initial plasma density $1\cdot10^{10}cm^{-3}$. In case of longitudinal injection of the plasma into the vacuum chamber, whose transverse dimensions are considerably less than relativistic Larmor radius of cyclotronic rotation, SGA yields a bunch of relativistic electrons (Fig. 2). Cyclotronic radius of this bunch is close to relativistic Larmor one, while its field is counterbalanced in part with the field of the ions confined by this field. Relatively narrow energy spectrum of the captured electrons and considerable excess of ions losses over the electrons losses are peculiarities of this case.

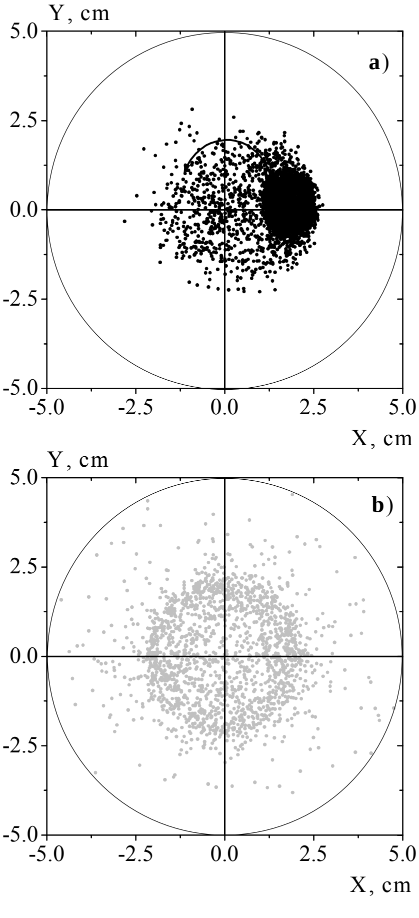

Fig.2 Cross section of the spatial distribution of the plasma particles at $r_o = 0.6$ *cm a) electrons; b) ions*

Another limiting case is shown in the Fig. 3. This case describes the SGA in the plasma-filled cavity or initial plasma creation by ECR-discharge. In this case SGA yields relativistic plasma entity, that remains quasineutral. Further on we shall denote this entity as SGA-plasma. Energy spectrum of captured electrons of SGA-plasma is significantly wider, than that of relativistic bunch. Besides, considerable part of entrapped electrons are not captured into the SGA mode. Ions have in this case virtually uniform spatial distribution (Fig. 3 b) in a contrast to the case with initial plasma radius $r_o = 0.6\ cm$, when electron field-entrapped ions form a ring (Fig. 2a).

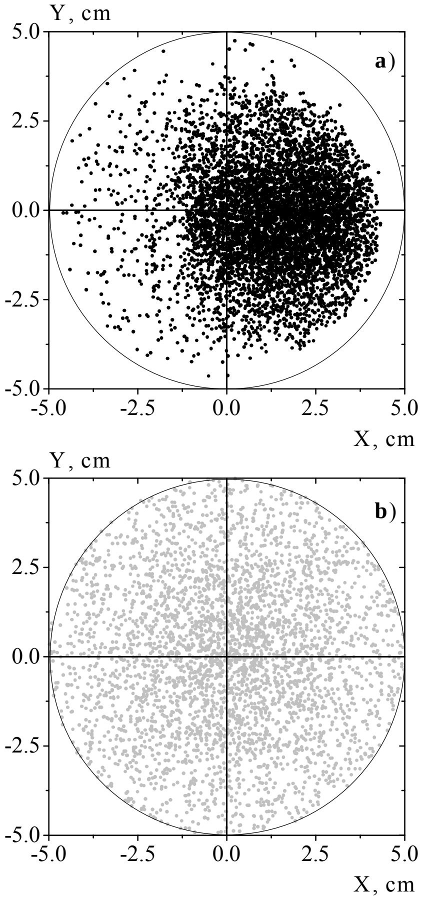

Fig.3 Cross section of the spatial distribution of the plasma particles at $r_o = 2.4\ cm$*: a) electrons; b) ions.*

One of the most important SGA features is the capture efficiency, that is determined by the ratio of the captured electrons number to the total initial number of plasma electrons N_{tr}/N_o. Capture efficiency dependence on the initial misalignment of magnetic field was studied at electric HF field strength of *3 kV/cm.* At the initial magnetic field equal to the resonance one, i.e., at $B(0)/B_o = 1$, 98% of the electrons remaining in plasma were captured into the SGA mode (Table 1). On the increase of the initial magnetic field up to $B(0)/B_o = 1.05$ and further to $B(0)/B_o = 1.10$, the number of electrons, captured into the SGA mode reduces (cf. Table 1). Further increase of $B(0)/B_o$ to 1.15 leads to almost complete absence of the electron capture into the SGA mode, although the number of entrapped particles N_c is as high as 90 %.

Table 1. Relative numbers of the captured and entrapped electrons.

$B(0)/B_0$	N_c/N_0	N_{tr}/N_0	N_{tr}/N_c
1.00	0.423	0.414	0.979
1.05	0.422	0.399	0.945
1.10	0.573	0.381	0.664
1.15	0.868	0.017	0.020

These results are in good accordance with the formula for the maximal value of initial magnetic field, where SGA mode is still possible [3].

$$B(0) = B_0\left(1 + 1.89g_0^{2/3}\right) \tag{3}$$

Electron capture and their confinement in the magnetic trap analysis has shown, that at SGA in purely electronic cloud the particles losses exceed electrons losses at SGA in hydrogen or argon plasma by 2.5–3 times (Fig. 4). Calculations shown in Fig. 4 were made for the following parameters: $E = 3$ *kV/cm,* mirror ratio $R = 1.2$; initial density $5\cdot10^9$ cm^{-3}.

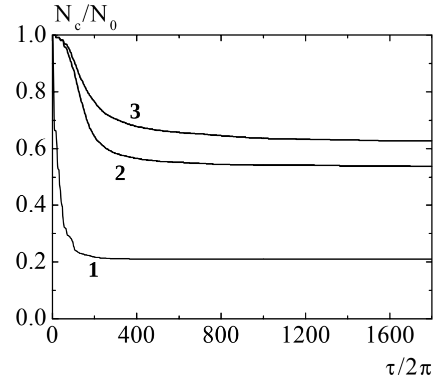

Fig.4. Relative number of entrapped electrons:
1. electronic cloud; 2. hydrogen plasma; 3. argon plasma

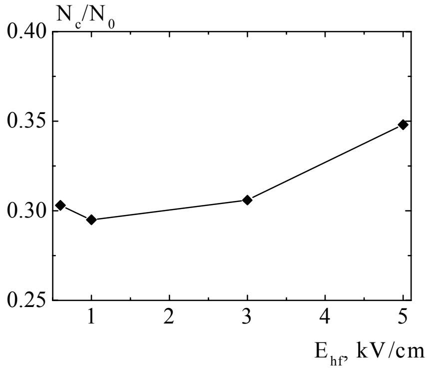

Fig. 5 Relative number of entrapped electrons

Electrons losses from plasma in case of heavier elements, e.g. of argon plasma, are lower than those from hydrogen plasma (Fig.4). This phenomenon is related to the higher mobility of the hydrogen ions as compared to argon ions. This means that to ensure efficient electron capture one should create initial plasma from the heavy gases.

Fig. 5 shows the dependence of electron capture on the electric strength of HF field E. Calculations were made at mirror ratio $R = 1.2$, initial argon plasma density $1\cdot10^{10}$ cm^{-3}, $r_o = 2.4$ cm. Obtained results indicate, that E change in the range from *600 V/cm* to *5 kV/cm* does not cause significant changes of capture efficiency, although some trend of capture efficiency to grow with increasing E is observed.

The studies of SGA-plasma parameters on the configuration of magnetic field in the trap were performed for two mirror ratios $R = 1.2$ and $R = 2.0$. Numeric experiments have shown, that at larger mirror ratio the density of captured particles increases. Thus, at electric field strength of *3 kV/cm* and initial density $1\cdot10^9$ cm^{-3} with mirror ratio $R = 1.2$ captured particles density is $4.9\cdot10^8$ cm^{-3}, while for the mirror ratio $R = 2$ captured electron ratio increases to $1.2\cdot10^9$ cm^{-3} (hydrogen plasma). This density growth is obtained (in spite of the particle losses) due to the axial compression of the plasma. With the mirror ratio growth increase both the numbers of captured (from $3.5\cdot10^9$ to $4.6\cdot10^9$) and entrapped particles (from $3.7\cdot10^9$ to $5.2\cdot10^9$). Therefore implementation of higher mirror ratio is one of the factors for optimization of generation of a bunch of relativistic electrons.

The results of numeric experiments have shown, that with the increase of the initial plasma density relative losses of the particles (cf. Fig. 6). Calculations were performed for the case of hydrogen plasma, $E = 3\ kV/cm$, $r_o = 0.6\ cm$, $R = 1.2$.

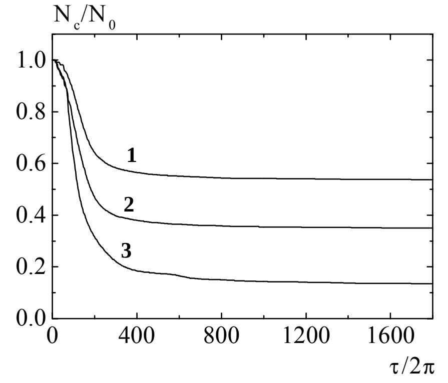

Fig. 6. Relative number of the entrapped electrons
1. $n_o = 5\cdot10^9 cm^{-3}$; *2.* $n_o = 1\cdot10^{10} cm^{-3}$; *3.* $n_o = 5\cdot10^{10} cm^{-3}$

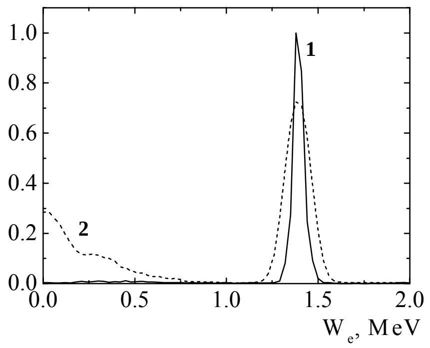

Fig. 7. Electron energy spectrum:
1. $n_o = 1\cdot10^{10} cm^{-3}$; *2.* $n_o = 5\cdot10^{10} cm^{-3}$

An important parameter describing SGA process is the width of electron energy spectrum. Present study deals with the dependence of the energy spectrum width on the initial plasma density, gas type, strength of HF electric field and mirror ratio of the magnetic trap.

Results of the numerical experiments, fulfilled for the case of hydrogen plasma ($E = 3\ kV/cm$, $r_o = 0.6\ cm$, mirror ratio $R = 1.2$), have shown, that with the growth of initial plasma density increases the relative number of electrons, not captured into SGA mode. Simultaneously particle losses from the plasma grow. This finds explanation in considerable influence of ambipolar field arising in plasma upon the particle dynamics. Energy spectrum of the captured electrons in this case broadens (Fig.7). The electron energy spectrum width depends also upon the strength of HF field (growth with increasing E) and transverse dimensions of initial plasma (Fig.8). Broadening of the captured electron energy spectrum in the latter case generally is due to the influence of non-uniformity of the magnetic field on the electron dynamics.

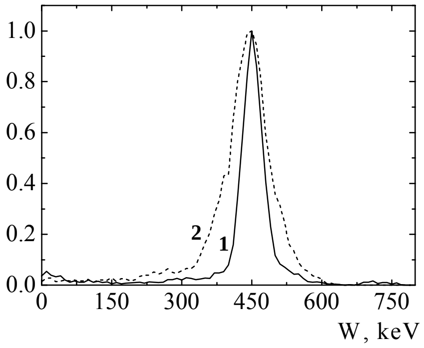

Fig. 8. Electron energy spectrum of argon plasma:
1. $r_o = 0.6$ cm; 2. $R_o = 2.4$ cm

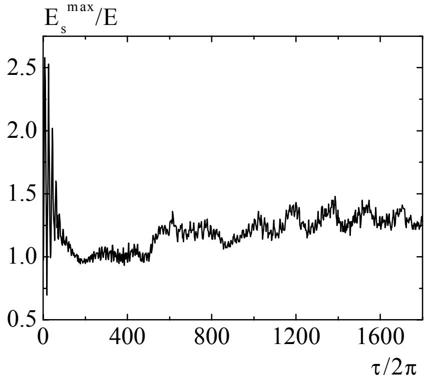

Fig. 9. Maximal value of the own electric field of the plasma

Time dependences of maximal magnitudes of the own electric field E^{max}_s, arising within the plasma permit to make a conclusion, that during the SGA in plasma occur low frequency oscillations, whose frequency for hydrogen plasma is 10 – 15 MHz, and for argon one 2-5 MHz. These oscillations fade away in time and the average E^{max}_s value approaches the amplitude of electric HF field. Simulation results indicate that low frequency oscillations are not growing and do not cause the plasma deterioration (Fig. 9), which is confirmed by laboratory tests [4]. The calculations shown in the Fig.9 are done for the hydrogen plasma with $R = 1.2$, $r_o = 0.6$ cm, $n_o = 1 \cdot 10^{10}$ cm^{-3}, $E = 3$ kV/cm.

3. Prospects of SGA applications

Tri-dimensional SGA simulation has shown, that its most characteristic feature is the possibility to obtain the entities with the parameters varying in the wide range from bunches of relativistic objects with narrow energy spectrum to plasma with relativistic electronic component. This allows various practical SGA applications.

With the small initial plasma radius ($r_o \leq 0.6$ cm) compact relativistic electron bunch is created with relatively small cross section of 1.5 − 2 cm and narrow energy spectrum. Such a bunch with electrons energy 500 keV − 5 MeV may be landed on the target to obtain bremsstrahlung radiation. The power of bremsstrahlung radiation may be from *1.2 kW* (with the landing time of *1μs*) to 1.2 MW (with landing time of *1 ns*) within 1 SGA-pulse with 10% electron kinetic energy conversion coefficient into bremsstrahlung radiation.

With initial plasma radius of the order of magnitude of relativistic Larmor radius arises the plasma with relativistic electronic component with broader energy spectrum, filling the larger part of the cavity. This plasma entity may be used, e.g., in the collective ion accelerator (presently being designed) ECRIPAC [7] after the stage of adiabatic compression.

4. Conclusion

Results of numeric simulation, performed in this paper, allow to conclude that:

1. SGA in the initial non-relativistic plasma allows to create both relativistic plasma and relativistic electron bunches. Parameters of relativistic electron bunches may be varied by the varying process parameters.

2. Numeric simulation allows to determine the optimal conditions for compact relativistic bunches generation. For instance, for mirror ratio $R = 1.2$ and $E = 3\ kV/cm$ initial plasma density should lie between $5 \cdot 10^9\ cm^{-3}$ and $1 \cdot 10^{10}\ cm^{-3}$, and its transverse dimension should not exceed *1.2 cm.*

3. Acceleration of the purely electronic cloud is not efficient due to the large electrons losses (losses grow more than three times as compared to plasma case).

4. SGA in the magnetic trap with the larger mirror ratio leads to the smaller losses and formation of the denser plasma due to the more pronounced effect of axial compression.

5. Acceleration of plasma electrons in the SGA mode is accompanied by appearance of low-frequency oscillations. These oscillations, however, are not growing and do not cause plasma deterioration.

6. SGA process is the most stable and electron capture efficiency into SGA mode grow for the cases of initial plasma with the ions of relatively heavy elements (e.g. argon) as compared to hydrogen plasma.

These conclusions and other results of numeric experiments, presented in this paper may be used in design of experimental Gyrac devices, oriented to practical use.

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